A METHODOLOGY FOR ROBUST DESIGN OF IMPINGEMENT COOLED HSCT COMBUSTOR LINERS

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Abstract

This paper describes the use of a robust design simulation methodology for the determination of an optimum lean, premixed, prevaporized combustor liner cooling configuration for a High Speed Civil Transport. The objective of design robustness as developed at the Georgia Institute of Technology is to find settings for design parameters which will not only maximize performance, but also minimize the influence of uncertainty on performance. This robust design simulation methodology is formulated here as a very general approach that lends itself to any design problem where uncertainty exists. This general methodology is applied to the design of a combustor liner in order to quantify the effect of cycle parameter and heat transfer coefficient uncertainties on combustor liner metal temperature variance. The results show that for the parameter ranges of interest, impingement hole spacing and thermal barrier coating thickness have the greatest effect on metal temperature variance and are used to find a robust liner configuration.

Introduction

Man has always sought ways to travel ever greater distances in less time than could be done previously. This desire to arrive at a destination faster than is currently possible has provided an impetus for the creation of a next-generation High Speed Civil Transport (HSCT). This aircraft is envisioned to be capable of cruising at supersonic speeds (Mach 2.4) and carry 300 passengers to destinations of up to 5,000 nmi according to NASA's goals for the program.

However, the desire for faster travel has been tempered by a growing environmental awareness which has raised concerns about the effect of a fleet of high-flying supersonic aircraft on the earth's stratosphere. In particular, emissions of Nitrogen Oxides (NO_x) are known to act as catalysts for the decomposition of ozone in the ozone layer. Emission of NO_x is of particular concern for an aircraft such as the HSCT because its cruising altitude is in the region where ozone concentration is highest, and consequently, most sensitive to pollution. As a result, any environmentally acceptable HSCT must emit low levels of NO_x.

The current target for emission levels is to produce only 5 grams of NO_x per kilogram of fuel consumed. This target is based on a series of studies conducted by government and industry,[1] and is approximately 1/10 the emissions levels of current subsonic aircraft. However, reducing emissions levels by an order of magnitude over current levels is rather ambitious and can only be achieved through the introduction of new "ultra-clean" combustion technology.

Several new low-emissions combustors are currently being studied by government and industry as candidates for use in the HSCT. Two of the most promising technologies are the rich-burn, quick-quench, lean-burn (RQL) and the lean, premixed, prevaporized (LPP) combustors. The central concept of both of these ultra-clean combustors is simple. Studies have shown that the formation of NO_x is primarily a function of flame temperature in the combustion region, and in general, higher combustion temperatures lead to greater concentrations of NO_x. Therefore, if one can achieve efficient combustion at lower temperatures, NO_x production will be curtailed.

The RQL and LPP combustors use different approaches to reduce combustion temperature. The RQL combustor uses a staged combustion technique while the LPP combustor depresses flame temperature by burning a lean fuel-air mixture, as shown in Figure 1. For the purposes of the current study, only the LPP configuration will be considered in detail, though the methodology developed here could be applied to analysis of RQL combustors as well.

The LPP combustor maintains a (relatively) low flame temperature by burning the fuel-air mixture at lower than stoichiometric (lean) conditions. In effect, the flow in the combustion region contains more air than is needed to completely combust all of the fuel in the combustion region. Thus, the remaining air acts as a diluent which cools the combustion byproducts to an intermediate temperature thereby inhibiting NO_x formation.

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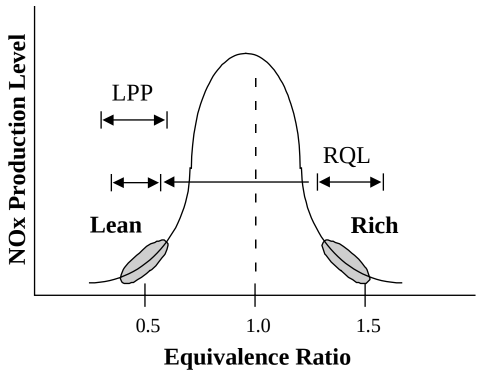

Figure 1: LPP and RQL Combustion Stoichiometry

Unfortunately, lean combustion creates difficulties in cooling the combustor internal parts, particularly the combustor liner. The reason is that in order to achieve lean combustion, one must *increase* the air flow rate passing through the combustion region, at the expense of the cooling flow. The reasoning can be explained as follows. The thrust performance of the engine equipped with an LPP combustor must be comparable to an engine equipped with a conventional combustor. This implies that power output (and fuel flow rate) must be the same for both configurations. However, the LPP configuration must have a lower fuel to air ratio than the conventional combustor. Since the fuel flow rates are the same, the LPP configuration must direct a larger percentage of the air flow into the combustion region, which results in decreased availability of cooling flow.

In a conventional combustor, approximately 50% of the compressor discharge flow is passed through the combustor dome and into the combustion region, while the remaining flow is bypassed around the combustion region and used for dilution and cooling. In the LPP configuration, most of the flow is passed into the combustion region, leaving very little for cooling and dilution.

Thus, the requirement for lowered NO_x emissions levels ultimately results in very demanding cooling requirements for the LPP combustor which can only be met through the use of new cooling technologies and design methods. Moreover, there is a great deal of uncertainty in combustor liner metal temperature predictions because metal temperatures are highly dependent on parameters that the designer has no control over. The result is that an LPP combustor will not have the luxury of a large design margin to ensure cooling effectiveness. This is the impetus for the application of robust design methods which can quantify the impact of uncertainty on combustor liner cooling characteristics and minimize their effects.

Uncertainty in Design

In the present context, design uncertainty is defined to be an inability to predict the outcome of an event or the exact value of a design parameter. Design uncertainty results from parameters that affect the performance of a system, but which the designer has no control over. Uncertainty is typically due to the nature of the problem (such as an accurate prediction of fuel cost as a function of time) or the dynamic nature of a system (such as the evolution of a design as it progresses from conceptual to detailed phases).

This idea of uncertainty provides a convenient means of grouping design parameters into two categories: *control parameters* and *noise parameters*. Control parameters are variables that the designer has direct control over, while noise parameters are variables that affect the design, but are beyond the designer's control. Control parameters can be specified deterministicly as a point value without loss in accuracy and can thus be optimized. However, since noise parameters are beyond the designer's control it does not make sense to set them at a single point value. Instead, *noise parameters should be specified probabilisticly in terms of a range and probability distribution*.

For example, fuel cost is a noise parameter that has a large effect on the operating cost of an aircraft, but is beyond the designer's control. Obviously, it is extremely difficult to predict fuel cost as a function of time. The best that one could reasonably hope to do is to define a range and probability distribution for fuel cost based on experience and use an analytical model to calculate a distribution for the aircraft operating cost.

This concept of design uncertainty is illustrated in Figure 2 which depicts the inherent uncertainty of noise parameters resulting in a response described in terms of a probability distribution. Almost all systems are subject to some form of uncertainty such as accuracy of design models, manufacturing tolerances, or environmental uncertainty, and thereby lend themselves to some form of robust design analysis.

Robust Design

A robust design in the sense used here is a design that is capable of operating efficiently in a wide variety of environments. The objective of robust design methodology is to account for the effect of design uncertainty by defining unknown parameters in terms of probability distributions and calculating design performance as a distribution of values. Design robustness is achieved by finding settings for control parameters which will not only maximize performance, but also minimize the influence of noise parameters on design performance[2].

A classic example of design robustness is the task of designing an aircraft to meet flyover noise levels legislated under Federal Aviation Regulations Part 36 (FAR36), which sets an upper limit on the allowable takeoff noise of aircraft. Flyover noise levels are effected by things such as wind direction, temperature, humidity, etc. However, these are all noise parameters which are beyond the designer's ability to control. The best that one can do is to define a range and probability

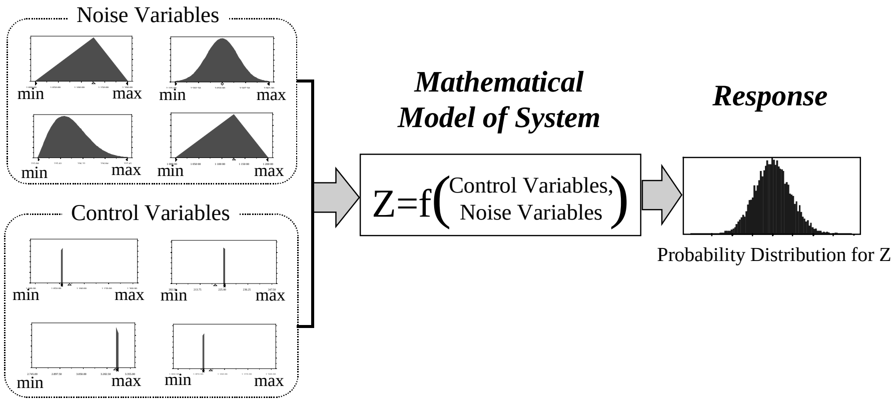

Figure 2: The effect of Uncertainty on Design

distribution for each of these parameters based on historical data. In turn, this information is used to calculate a probability distribution for flyover noise using some type of analytical model. However, the fact that these parameters are beyond the designer's control does not mean that a designer cannot consider reducing the effects of these variables. For example, a design can be made more robust by employing a new technology such as variable cycle engines (VCEs) which give the designer the ability to tailor the engine cycle to compensate for these uncertainty factors and maximize performance while remaining within noise regulations.

From an aircraft operator's viewpoint the flyover noise must not only be as low as possible, but should also have a minimal variance. In effect, one would like to have some assurance that an aircraft which meets FAR36 on a rainy day in Los Angeles will still be within regulations on a hot Denver day. Yet at the same time, the aircraft cannot be over-designed if it is to be profitable to operate. The goal of robust design is to *minimize the influence of uncertainties* such as atmospheric conditions on the performance of the design while guaranteeing an affordable solution.

A similar situation arises in combustor liner cooling design. Several factors that are important to the combustor liner temperature are noise parameters. Robust design methods are needed to quantify uncertainties and develop a configuration that will not exceed material temperature limitations while simultaneously avoiding over-design of the cooling configuration.

Robust Liner Design Methodology

The objective of liner cooling design is to keep the liner peak metal temperature below some maximum metal temperature, set by strength and material considerations, using the smallest cooling mass flow rate possible (material thermal stress considerations are not considered at this point). In addition, since an LPP combustor must use minimal liner cooling flow, there is little design margin available to compensate for uncertainty in the analysis process. Thus, the designer needs a way of minimizing the effect of uncertainty in such a manner as to be reasonably assured that liner temperature limitations will not be exceeded regardless of flight condition, ambient temperature, manufacturing imperfections, and so on. Robust design methods provide a way to account for these uncertainties and minimize their effects without unnecessarily penalizing the configuration.

Liner Cooling Model

A step-by-step description of robust cooling design methodology is given in Figure 3. The central element is an analysis tool that is capable of predicting the peak liner metal temperature as a function of cooling geometry, liner flame (gas) side boundary conditions, and liner backside (coolant-side) boundary conditions (depicted inside the dashed box in Figure 3).

Backside boundary conditions are calculated through cycle analysis and cooling analysis. Cycle analysis is used to calculate coolant temperatures (T_3) while cooling analysis is used to calculate backside heat transfer coefficient. Typically, this cooling analysis is based on a regression of experimental data for the cooling configuration under consideration.

Flame-side (or gas side) boundary conditions are more difficult to calculate because flame side flow patterns are usually not uniform. Those zones near a swirler cup are subject to intense flame scrubbing which creates hot-spots in the liner material. In addition, the calculation of radiative heat flux into the liner walls is a very difficult task and typically consists of a considerable amount of guesswork on the designer's part. Finally, it is usually necessary to use CFD analysis in order to get a reasonably accurate estimation of the boundary conditions along the wall. Lacking this, one must at least have some idea based on historical knowledge of what typical values for convective heat transfer coefficient and adiabatic wall temperature are for the configuration under consideration.

The liner thermal model and boundary condition calculation routines collectively constitute a mathematical model for liner metal temperature as a function of cycle parameters and cooling system geometry. In an

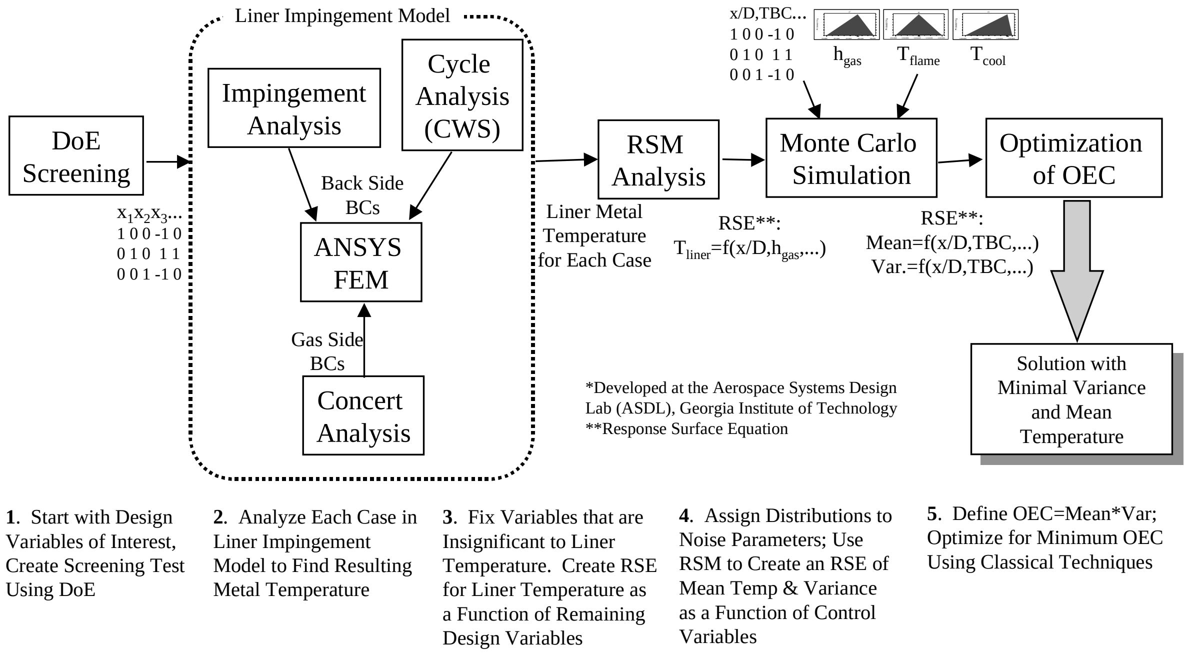

Figure 3: Robust Combustor Liner Cooling Design Methodology

abstract sense, one can think of the liner cooling model as a “black box” which takes cooling geometry and cycle information and returns a liner metal temperature, as shown in Figure 3. Since several of the input parameters may be considered as noise factors, the model has some inherent uncertainty in the calculation of liner temperature, as described in Figure 2.

Response Surface Analysis

The best way to calculate a stochastic response distribution is through Monte Carlo simulation. Unfortunately, Monte Carlo simulation requires thousands of trials to get a reasonably accurate approximation for the response distribution, and running 10,000 trials using a finite element model would be prohibitively time consuming. This is especially true in light of the fact that the liner cooling model described previously is rather complex and has numerous inputs. Therefore, a simplified representation of the liner model is needed which can be quickly run to get an accurate estimate for liner temperature.

The Response Surface Methodology (RSM) formulation provides an analytical method for creating this simplified model. It uses design of experiments (DoE) to create a set of experiments (or simulations) which are run to find a response for liner temperature. These simulations are then used to derive a polynomial equation for liner metal temperature as a function of liner geometry and engine cycle variables. This Response Surface Equation (RSE) can then be used in place of the detailed liner model to perform the Monte Carlo simulation.

The first step in the RSM analysis is to conduct a screening test to eliminate those variables which do not contribute significantly to the response. The concept of a screening test is based on Pareto analysis[3], which seeks to identify and prioritize the contributing variables in order of importance. In this case, the most significant contributors can be identified and kept for further development while the remaining variables can be set at their most likely values. The screening test is essentially a sensitivity analysis used to identify those few variables that are truly important. This process is shown in steps 1, 2, and 3 of Figure 3. The net result is a reduction in the number of cases that must be run to obtain the RSE.

A byproduct of the screening process is a gain in knowledge about the relative importance of each factor. This gives the designer a feel for which variables are the most important and by how much. Alternatively, if the designer is familiar with the relative importance of each variable, the screening test can be used to spot mistakes or unusual results in the data. If the screening test results agree with past design experience, then the designer has some degree of assurance that the analysis is correct.

Having fixed the unimportant variables to their most likely values, the next step is to use RSM to create an equation for liner temperature as a function of the important variables identified in the screening test. This entails running another set of cases, varying only the important factors (shown in step 3 of Figure 3). The results of these cases are then used to generate an RSE for liner temperature as a function of the most important variables.

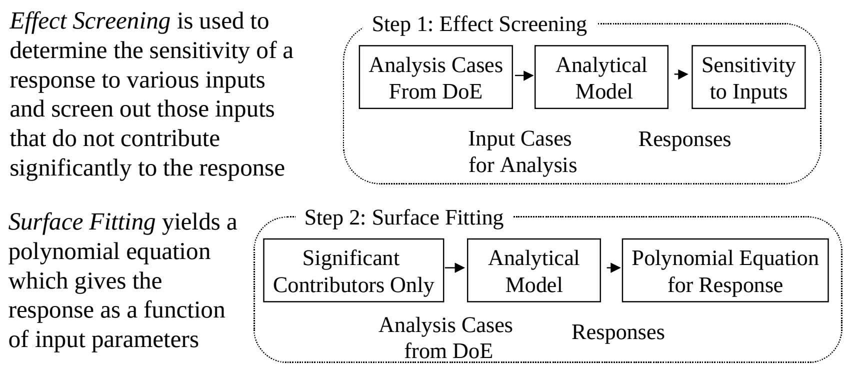

Figure 4: Response Surface Methodology Flowchart

This two-step process is illustrated in Figure 4. The "analytical model" mentioned in this figure is the liner impingement model shown in the dashed box of Figure 3. Essentially, DoE and RSM are statistical methods that are used to create multivariate regression equations for responses in terms of input parameters, using a minimal number of experiments (or cases). These methods have been used in industrial and systems engineering for decades and are finding increased use in the aerospace industry. Numerous texts have been published on these subjects and therefore, they will not be discussed in detail here. Instead, the reader is encouraged to consult References 4, 5, and 6 for further information on these topics.

<u>*Monte Carlo Simulation*</u>

The next step is to incorporate the effect of uncertainty into the liner temperature model. This is accomplished using a Monte Carlo simulation, as shown in Figure 3, step 4. Monte Carlo simulation is essentially a tool which calculates a response distribution (temperature) based on input distributions (noise variables) and fixed values of control parameters. Previously, RSM was used to derive an RSE for liner temperature which was a function of both control and noise variables. Since the noise variables are now described in terms of distributions, the metal temperature must also be a distribution as well.

This response distribution can be described in terms of a mean and variance. In order to minimize the mean and variance of a distribution, one must first derive analytical expressions for mean and variance in terms of the control variables. This is done using RSM in a similar fashion as previously by assigning fixed distributions to the noise parameters and using DoE to create another set of cases with *only the control factors as variables*. Each case is then run in the Monte Carlo simulator to find a mean temperature and variance. This data is then used to create RSEs for liner temperature mean and variance as a function of control factors only.

<u>*OEC Optimization*</u>

The mean and variance equations are effectively a parametric description of the response distribution in terms of control factors which the designer can directly manipulate. These equations can be optimized for minimal mean and variance subject to constraints on allowable metal temperature and cooling flow rate using canned optimization methods (step 5, Figure 3). The most convenient way of concurrently optimizing both mean and variance is to create an overall evaluation criterion (OEC) of the form:

$$\text{OEC}=\text{Mean}*\text{Variance}$$

Minimization of this OEC leads to a solution with minimal variance and mean temperature.

<u>LPP Combustor Liner Robust Design</u>

The combustor configuration considered in the HSCT robust liner design analysis is an impingement cooled shingled combustor liner. The basic layout of the LPP combustor configuration is shown in Figure 5. The liner consists of three segments (or shingles) each of which is impingement cooled. Combustion air passes through the premixer into the flame region while cooling air passes into the inner and outer passages and through the impingement baffle. After impinging on the liner backside, the spent air is exhausted into the high pressure turbine gas stream.

Impingement cooling is currently the method of choice for LPP combustor cooling purposes because of its high cooling effectiveness and low coolant mass flow rate. Impingement cooling uses a perforated plate (an impingement baffle) to direct jets of cooling air onto the liner back-side as shown in Figure 5. The result is a high average convective heat transfer coefficient relative to other methods of liner cooling.

Impingement cooling analysis methods are primarily based on experimental data. The reason for this is that the flowfield in the impingement gap is highly turbulent and is not well understood, making it very difficult to create an analytical model. Instead, efforts

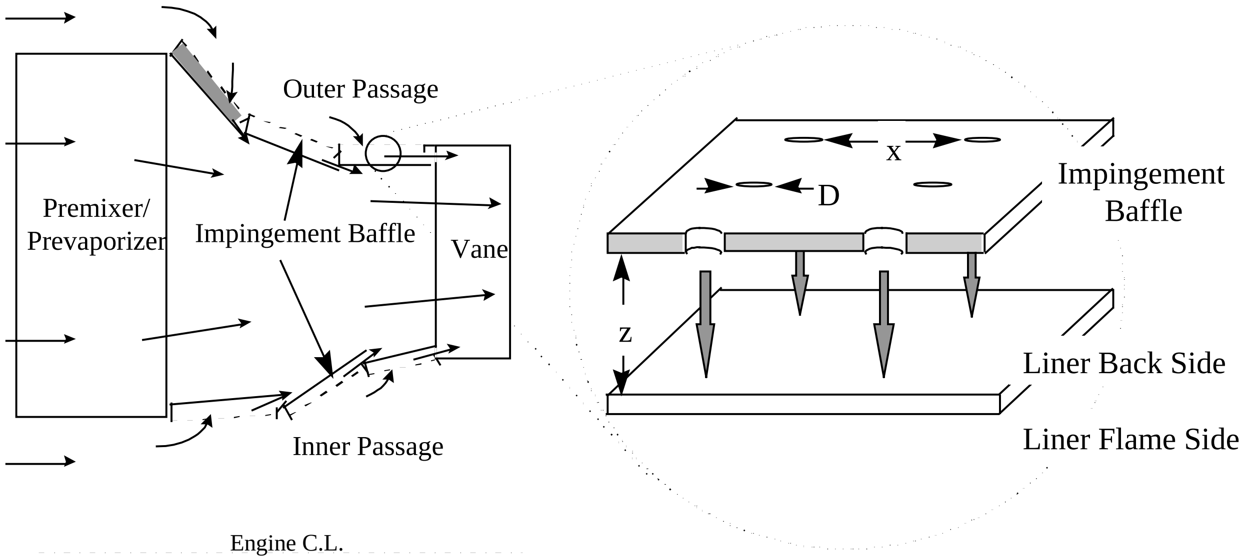

Figure 5: LPP Combustor Liner Configuration

have focused on experimental work to correlate backside heat transfer coefficient with impingement geometry and pressure drop.

One of the best known correlations for impingement heat transfer coefficient is that of Kercher and Tabakoff[7]. In this paper, the researchers derive a regression equation for average heat transfer coefficient in terms of hole diameter Reynolds number (Re_D), hole diameter (D), nondimensional hole spacing (x/D), gap spacing (Z/D), and flow crossflow. This correlation is used to calculate backside heat transfer coefficients for all analysis cases investigated.

Analysis Assumptions

For the current study, several simplifying assumptions are needed to limit the scope of the current investigation to a size reasonable for explanation in a single paper. These assumptions are based on previous analysis of similar combustor configurations and are:

- Only the first shingle (shaded in Figure 4) is considered for analysis because CFD studies indicate that it is subject to the worst flame scrubbing conditions.
- Gap crossflow due to spent impingement air is assumed to have a negligible effect on the backside heat transfer coefficient for the configuration considered. Given the short length of each shingle, this is a reasonable approximation.
- The pressure gradient inside the gap is assumed to be negligible. Coolant mass flow is therefore uniform over the impingement baffle area.
- The impingement hole discharge coefficient is 0.8.
- The flight condition for all cases is top-of-climb (Mach 2.4, 50,000 ft).

Finite Element Model

The finite element model used to calculate maximum metal temperatures consists of a small piece of combustor liner located between two impingement points. This is modeled using 2-D plane thermal elements in ANSYS 5.1[8] as shown in Figure 6. This model is composed of 166 elements and includes flame radiation and convective heat transfer effects.

The model consists of three layers of material with a metal substrate on the coolant side and thermal barrier coating (TBC) material on the flame side. These layers are separated by a thin layer of bond coat which is used to bond the TBC to the metal. Obviously, the peak metal temperatures will occur on the side nearest the flame, and thus, the node temperature at the metal-bond coat interface is of interest.

The flame and gas-side boundary conditions calculated from cooling analysis, CFD, and cycle analysis are specified as a uniform convection coefficient and temperature on the top and bottom of the model. The sides are assumed to be adiabatic, therefore, this finite element model is effectively a 1-D thermal model for liner temperature.

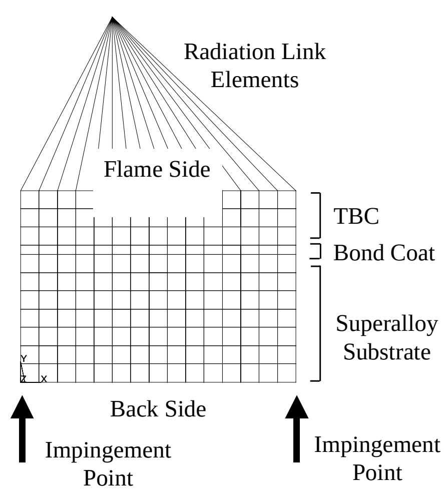

Figure 6: ANSYS Finite Element Model

Parameter	Units	Control/Noise
x/D	-	C
Z/D	-	C
D	in	C
T_{flame}	F	N
T_{cool}	F	N
h_{gas}	BTU/hr-ft2-F	N
K	BTU/ft-hr	C
ΔP/P	% P4	C
TBC Thickness	in	C

Table 1: Screening Test Parameters and Ranges

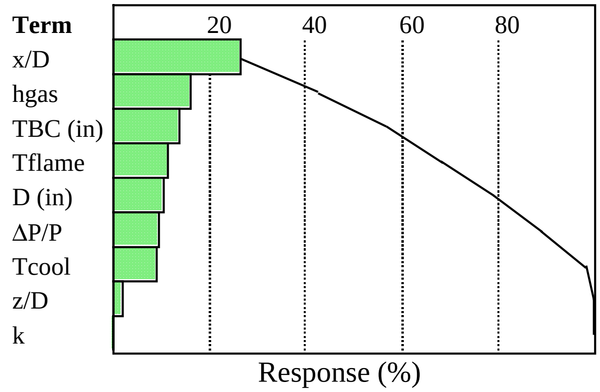

Figure 7: Pareto Plot of Liner Cooling Parameters

Screening Test

The impingement model previously described in the "liner cooling model" subsection consists of a set of nine input parameters and a single response, listed in Table 1. Three of these are impingement geometry parameters (x/D, Z/D, D) and two are engine cycle parameters (T_{flame}, T_{cool}). x/D is the non-dimensional impingement hole spacing, Z/D is impingement gap spacing, and D is impingement hole diameter. The parameter h_{gas} is the flame side convective coefficient, K is liner metal thermal conductivity, ΔP/P is impingement baffle pressure drop as a percentage of P_4, and TBC Thickness indicates the thickness of the TBC coat on the flame side of the liner. T_{flame} is the adiabatic flame temperature, and T_{cool} is the coolant temperature impinging on the liner backside (taken to be equal to compressor discharge temperature). Note that the design variables consist of three noise (denoted "**N**") and six control parameters (denoted "**C**").

A minimum and maximum value was selected for each of the variables based on design experience, and a DoE used to set up a fractional factorial experiment consisting of 33 cases. For all cases, the value of each input parameter was set either to the max (+1) or min (-1) value with the exception of the last case, in which all inputs were set to the midpoint of each range. Due to the proprietary nature of these ranges, only the normalized (±1) values for each variable are presented here.

Each case was run using the liner cooling model defined earlier to find the peak metal temperature in the liner. This data was then analyzed using a statistical analysis package called JMP[9]. The results can be expressed in the form of a Pareto plot as shown in Figure 7.

The Pareto plot is a visual tool used to show the relative importance of several parameters by ranking the parameters in order of decreasing importance. The line to the right of the bars gives the cumulative contribution for each of the parameters as a percentage of the overall response. The Pareto plot shown in Figure 7 shows that the hole spacing, x/D, contributes 27% of the response while the metal conductivity and gap spacing contribute only 3-5%. Since the latter two parameters are insignificant to liner temperature, they can be fixed at their midpoint values without sacrificing RSE model accuracy. In effect, the number of variables is now reduced from nine to seven. It is important to note that the response is strongly proportional to the variable ranges selected and it is important to select reasonable ranges for each parameter.

The next step is to use DoE to create another series of cases with which to create an RSE. Again, these cases were executed using the liner cooling model and the data was analyzed using JMP. The result is an equation for liner temperature as a function of the seven remaining variables which effectively takes the place of the complicated analytical impingement model. This RSE is a quadratic polynomial equation in seven variables of the form:

$$R = b_0 + \sum_{i=1}^{k} b_i x_i + \sum_{i=1}^{k} b_{ii} x_i^2 + \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} b_{ij} x_i x_j$$

where b_0 is the intercept, x_i denotes a design variable, b_{ij} are the polynomial coefficients derived from regression analysis, and k=7 (for this case). This RSE is quite accurate (R^2 = 99%) and can predict the liner temperature to within a few degrees of the value calculated using the analytical cooling model.

Monte Carlo Simulation

As mentioned earlier, the design is subject to uncertainty in several of the input parameters. For instance, the coolant temperature (T_{cool}) is beyond the combustor cooling designer's ability to control and usually changes somewhat as the design evolves (it also changes with flight condition, engine wear, etc.). Similarly, the adiabatic flame temperature on the gas side is set by engine cycle considerations. Finally, the gas-side heat transfer coefficient (h_{gas}) is subject to considerable uncertainty due primarily to the complexity of the flowfield.

The next step is to use Monte Carlo simulation to show the effect of uncertainty on the liner temperature. In order to do this, each of the noise parameters is given

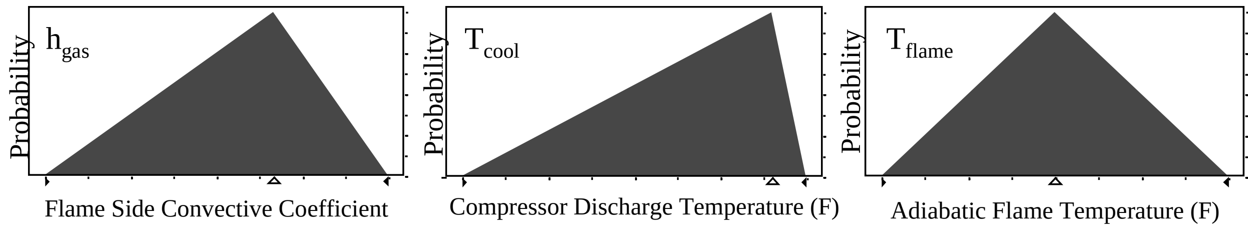

Figure 8: Probability Distributions for Noise Parameters

a fixed probability distribution based on design experience (shown in Figure 8). Note that each of the three distributions is described using a triangular probability shape. The triangular shape was selected because very little is known about the true shape of the probability distribution except the minimum, maximum, and most likely values. Thus, the triangular distribution is treated as a first approximation.

As an aside, it is interesting to note here that the classification of a parameter as a control or a noise variable depends largely on one's point of view. From a combustor cooling designer's point of view, T_{cool} is a noise parameter because it is set by cycle design considerations. However, from a cycle design perspective, this parameter is most definitely a control factor (T_3). Thus, one designer's noise parameter can be another's control parameter.

After assigning fixed distributions to the noise parameters, it is possible to assess how the control parameters affect the mean and variance by simply changing the values for the control parameters and looking at the resultant mean and variance. Equations for liner temperature mean and variance are derived using RSM in the same fashion as for the liner temperature RSE before. Once again, DoE is used to design an experimental setup consisting of 27 cases in which each of the four control variables is varied between their minimum and maximum values.

Each of these cases is passed into a Monte Carlo simulation package called Crystal Ball[10] to get a distribution for liner metal temperature. The mean and variance of each response distribution are recorded and RSM is used to derive two response equations: one for liner temperature mean and one for temperature variance as a function of the control parameters x/D, D, ΔP/P, and TBC thickness. These equations are quadratic polynomials in four variables instead of seven as was the case for the previous liner temperature equation. Once again, the model fit for these equations is very good, with the R^2 value for mean temperature being 99.9948% and the corresponding value for variance being 99.1467%.

Statistical analysis of the Monte Carlo simulation results shows that the hole spacing (x/D) and the TBC thickness have a strong effect on the temperature variance while the effect of pressure drop and hole size are small. Similarly, TBC thickness and hole spacing have a strong effect on liner mean temperature. This trend is reflected in the Pareto plots of Figures 9 and 10. Note that the liner temperature variance is dominated by the first two terms, whereas the mean temperature must include at least the first 4 terms in order to get a reasonable approximation of the response. Also, note that the interaction of x/D with itself is a significant factor in the determination of liner mean temperature.

These results can also be given in the form of prediction profiles as shown in Figure 11. The prediction profile is a matrix of plots showing each variable plotted against each response to show how the responses are effected by each parameter over the ranges investigated. The prediction profiles also give the designer an estimate of response sensitivities. A steep profile indicates the response is highly sensitive to that input, while a flat line indicates the input has no effect on the response.

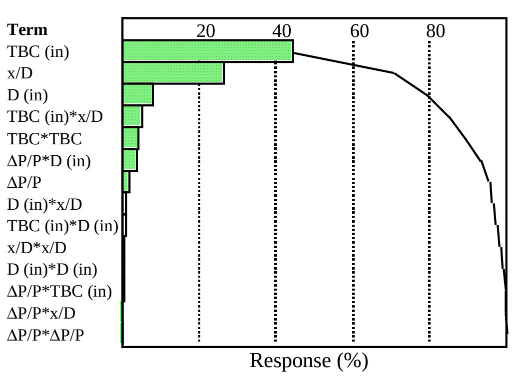

Figure 10: Pareto Plot of Combustor Liner Temperature Variance RSE

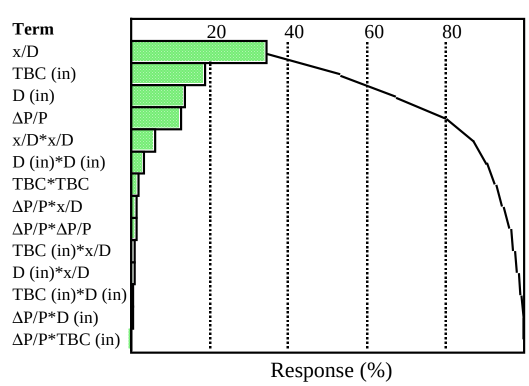

Figure 9: Pareto Plot of Combustor Liner Mean Temperature RSE

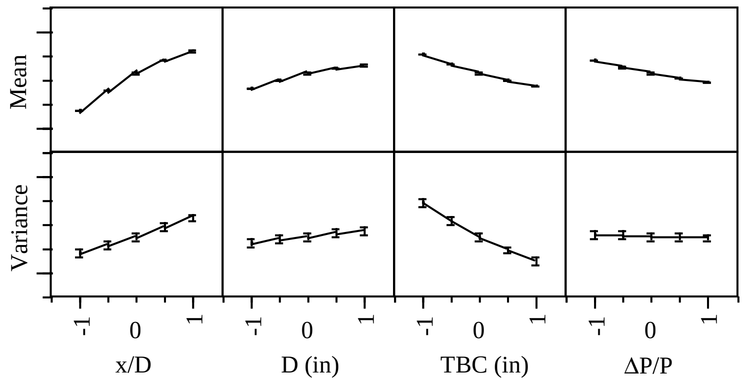

Figure 11: Prediction Profiles for Combustor Liner Mean Temperature and Variance

OEC Creation and Optimization

As stated earlier, the objective of robust design is to deliver a product that exhibits both optimum performance and minimal variance by formulating an OEC which captures the effect of both mean and variance. This OEC is easily created by multiplying the mean and variance of each case to form a new response. This response can then be analyzed to create an RSE for the OEC as a function of the four control variables.

The results of the OEC analysis are shown in Figures 12 and 13. Figure 12 shows prediction profiles for the OEC equation. The slope of the lines for hole spacing and TBC thickness are relatively steep indicating that they contribute significantly to the overall response.

The Pareto plot for the OEC equation in Figure 13 indicates a similar result. This figure indicates that the TBC thickness and hole spacing are by far the most dominant parameters. This result is expected based on the results of the separate analysis of mean and variance. These plots show the designer which parameters are important to the design in a very visual and intuitive way. Furthermore, if the analysis results are in error, it will usually be immediately obvious from the trends in the prediction profiles and Pareto plots.

Based on the current analysis, one can conclude that the combustor liner cooling designer should be judicious in the selection of TBC thickness and hole spacing, since these parameters have a drastic effect on liner temperature mean and variance. This result that may not be immediately obvious using other analysis methods and is one of the greatest strengths of the robust design method advocated here.

Since the OEC equation is a simple quadratic polynomial equation, it is well-behaved, has only one extremum, and can be optimized for minimum metal temperature using simple optimization techniques. However, in the case of the current analysis it is not necessary to go to such lengths; rather, one need only examine the prediction profiles. Figure 12 indicates that the lowest liner temperature will occur for the minimum hole spacing (x/D=-1), maximum TBC thickness (+1), maximum impingement baffle pressure drop (+1), and minimum hole diameter (-1). This result agrees well with the intuitive expectation that the minimal temperature occurs when each of these parameters are set to their minimal values. However, for problems in which the variables are tightly coupled and have strong interactions with one another, the parameter settings for optimum performance with minimal variance are not so obvious as in this example, and one must resort to classical optimization methods.

The improvement of the robust configuration with respect to the baseline configuration is shown in Figure 14. For the baseline case, the control parameters are set to the midpoint of their ranges, resulting in the liner temperature distribution on the right. The probability distributions assumed for both cases are shown in Figure 8. Note that the control parameters have a drastic effect on the mean of the distribution, while the effect on variance is less pronounced but still discernible.

Up to this point, the optimization has been unconstrained with respect to coolant mass flow rate. If the problem were constrained to have a fixed coolant

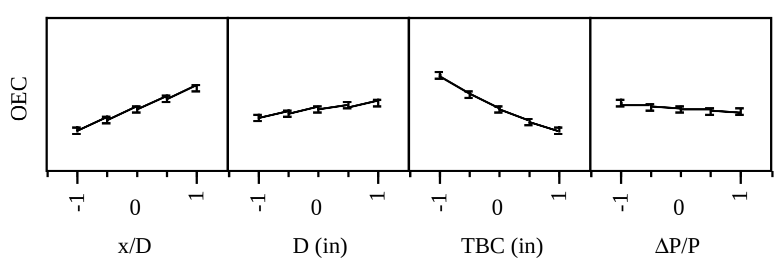

Figure 12: Prediction Profiles for OEC Optimization Analysis

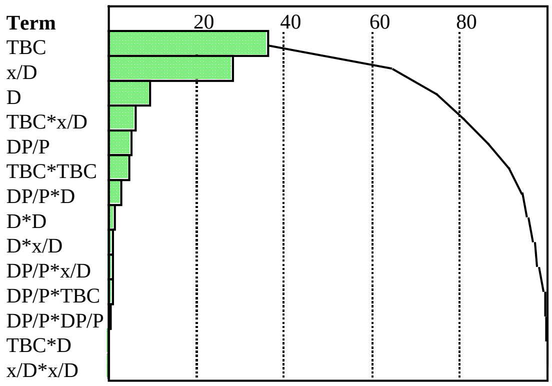

Figure 13: Pareto Plot of OEC Equation Results

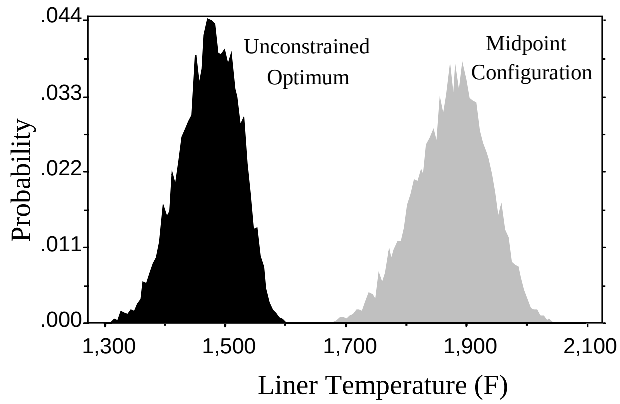

Figure 14: Comparison of Baseline (Right) and Robust Liner Temperature Distributions

mass flow rate and maximum liner metal temperature, one could not pick values for hole spacing and impingement baffle pressure drop independently. Instead, an analytical expression for coolant mass flow rate must be derived in terms of hole spacing and pressure drop. It is then possible to use a constrained analysis to find the best configuration that does not violate constraints.

Constrained optimization problems such as the one under consideration are usually expressed in terms of a design plot. From the impingement designer's point of view, the most sensible plot to show is hole spacing versus hole diameter. In impingement cooling design, there are two principal constraints: one for mass flow rate and the other for liner metal temperature. It is desirable to find a design point which has the fewest impingement holes and largest hole diameter possible because this reduces production cost of the liner.

Since the mass flow rate is a function of control parameters, it is a simple matter to fix pressure drop and plot a line of constant mass flow rate on the design plot. However, liner temperature is uncertain and can only be expressed in terms of a probability distribution. Therefore, a constraint on liner temperature is no longer a line and is instead a band of distributions. A detailed consideration of this intriguing possibility is beyond the scope of the current text and is left for future consideration.

Conclusions

The objective of this paper has been to detail a robust combustor liner design method to assist the designer in finding combustor configurations capable of meeting stringent cooling requirements in a system where there is little design margin. The Robust Design Simulation is a very general method and lends itself to any design problem where uncertainty is an important factor. In this case, the method was applied to an HSCT LPP combustor to show that:

- Given the choice between a large hole spacing with high pressure drop and a small spacing with a low pressure drop, the latter provides a more robust configuration with respect to noise.
- Increasing TBC thickness and decreasing hole spacing tend to decrease design variance.
- Liner metal conductivity and impingement gap spacing have little effect on liner temperature mean or variance.
- Finding new processes for increasing maximum TBC thickness capability has large potential payoffs because this parameter not only reduces liner temperature, but also reduces temperature variance.

The basic impingement cooling model developed in this paper is quite general and is also relatively simple. Thus, it lends itself to further development and more detailed analysis. Some future possibilities for improving the model in this study include adding a more accurate liner cooling model and incorporating economic aspects to find a robust configuration that has minimal manufacturing cost. One could also incorporate risk/benefit tradeoff studies to show the improvement in product performance with the introduction of new technologies and show which of these technologies has the greatest benefit to risk ratio.

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